Analog & Digital Control System Design

Transfer-function, State-space, & Algebraic Methods



CHEN

Analog and Digital

Control System Design:

Transfer-Function, State-Space, and Algebraic Methods

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1 Introduction

1.1 EMPIRICAL AND ANALYTICAL METHODS

The ultimate goal of engineering—in particular, that of control engineering—is to design and build real physical systems to perform given tasks. For example, an engineer might be asked to design and install a heat exchanger to control the temperature and humidity of a large building. This has been a longstanding problem in engineering, and much relevant data has been collected. From the total volume and geographical location of the building, we can determine the required capacity of the exchanger and then proceed to install the system. If, after installation, the exchanger is found to be insufficiently powerful to control the building's environment, it can be replaced by a more powerful one. This approach, which relies heavily on past experience and repeated experimentation, is called the *empirical method.* Although the empirical method must be carried out by trial and error, it has been used successfully to design many physical systems.

The empirical method, however, is inadequate if there is no past experience to draw from or if experimentation is not feasible because of high cost or risk. For example, the task of sending astronauts to the moon and bringing them back safely could not have been carried out by the empirical method. Similarly, the design of fusion control in nuclear power plants should not be so dealt with. In these cases, the analytical method becomes indispensable. The *analytical method* generally consists of four steps: modeling, setting up mathematical equations, analysis, and design. The first two steps are closely related. If we use simple mathematics, then the model chosen must be correspondingly simple. If we use sophisticated mathematics, then

the model can be more complex and realistic. Modeling is the most critical step in analytical design. If a physical system is incorrectly modeled, subsequent study will be useless. Once a model is chosen, the rest of the analytical design is essentially a mathematical problem.

Repeated experimentation is indispensable in the empirical method. It is also important in the analytical method. In the former, experiments must be carried out using physical devices, which might be expensive and dangerous. In the latter, however, experiments can be carried out using models or mathematical equations. Many computer-aided design packages are available. We may use any of them to simulate the equations on a digital computer, to carry out design, and to test the result on the computer. If the result is not satisfactory, we repeat the design. Only after a design is found to be satisfactory, will we implement it using physical devices.

If the model is adequately chosen, the performance of the implemented system should resemble the performance predicted by analytical design or computer simulation. However, because of unavoidable inaccuracy in modeling, discrepancies often exist between the performance of the implemented physical system and that predicted by the analytical method. Therefore, the performance of physical systems can often be improved by fine adjustments or tunings. This is why a physical system often requires lengthy testing after it is implemented, before it is put into actual operation or mass production. In this sense, experience is also important in the analytical method.

In the analytical approach, experimentation is needed to set up models, and experience is needed (due to the inaccuracy of modeling) to improve the performance of actual physical systems. Thus, experience and experimentation are both used in the empirical and analytical approaches. The major difference between these two approaches is that in the latter, we gain, through modeling, understanding and insight into the structure of systems. The analytical approach also provides systematic procedures for designing systems and reduces the likelihood of designing flawed or disastrous systems.

In this text, we study analytical methods in the analysis and design of control systems.

1.2 CONTROL SYSTEMS

This text is concerned with the analysis and design of control systems; therefore, it is pertinent to discuss first what control systems are. Before giving a formal definition, we discuss a number of examples.

1.2.1 Position Control Systems

The satellite dish in the backyard or on the rooftop of a house has become common in recent years. It is an antenna aimed at a satellite that is stationary with respect to the earth and is used to transmit television or other signals. To increase the number of channels for a television, the dish may be designed to aim at different satellites.

A possible arrangement of such a system is shown in Figure 1.1(a). This system can indeed be designed using the empirical method. If it is to be designed using the analytical method, we must first develop a model for the system, as shown in Figure 1.1(b). The model actually consists of a number of blocks.[1] Each block represents a model of a physical device. Using this model, we can then carry out the design. A large number of systems can be similarly modeled. For example, the system that aims the antennas shown in Figure 1.2 at communication satellites and the systems that control various antennas and solar panels shown in Figure 1.3 can be similarly modeled. These types of systems are called *position control systems.*

There are other types of position control systems. Consider the simplified nuclear power plant shown in Figure 1.4. The intensity of the reaction inside the reactor (and, consequently, the amount of heat generated) is controlled by the vertical position of the control rods. The more deeply the control rods are submerged, the more heat the reactor will generate. There are many other control systems in the nuclear power plant. Maintenance of the boiler's water level and maintenance or regulation of the generated voltage at a fixed voltage all call for control systems. Position control is also needed in the numerical control of machine tools. For example, it is possible to program a machine tool so that it will automatically drill a number of holes, as shown in Figure 1.5.

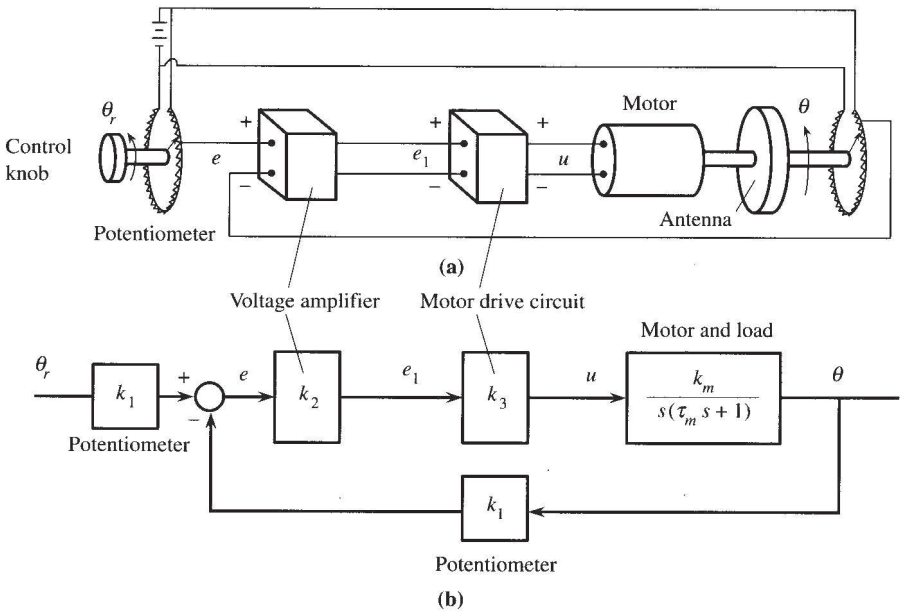


Figure 1.1 Position control system.

[1]At this point, the reader need not be concerned with the equation inside each block. It will be developed in Chapter 3.

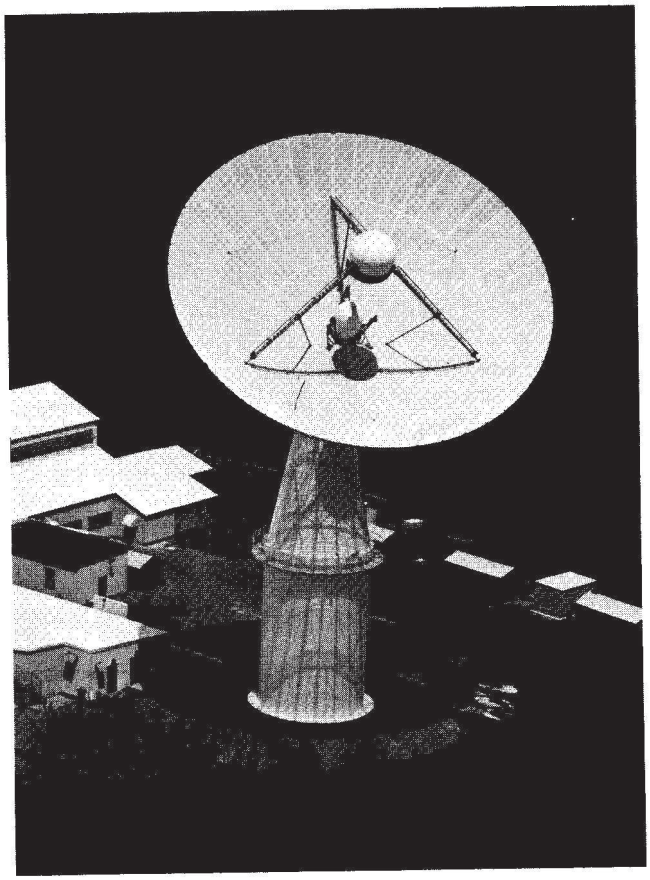


Figure 1.2 Radar antenna. (Courtesy of MIT Lincoln Laboratory.)

1.2.2 Velocity Control Systems

Driving tapes in video or audio recorders at a constant speed is important in producing quality output. The problem is complicated because the load of the driver varies from a full reel to an empty reel. A possible control system to achieve this is shown in Figure 1.6(a). This system can indeed be designed using the empirical method. If it is to be designed using the analytical method, we must develop a model, as shown in Figure 1.6(b). Using this model, we can then carry out the design. Velocity control problems also arise in a large number of industrial applications. To "grow" an optical fiber with a uniform diameter from melted glass, the speed of growth must be properly controlled. The speed of the conveyor in a production line must also be precisely controlled. An error in roller speed of just 0.1% in a paper drive system may cause the paper to tear or to pile up on the roller. The velocity of the rotor of the generator in Figure 1.4 is kept constant in order to generate a constant voltage. Velocity control is indeed needed in a wide range of applications.

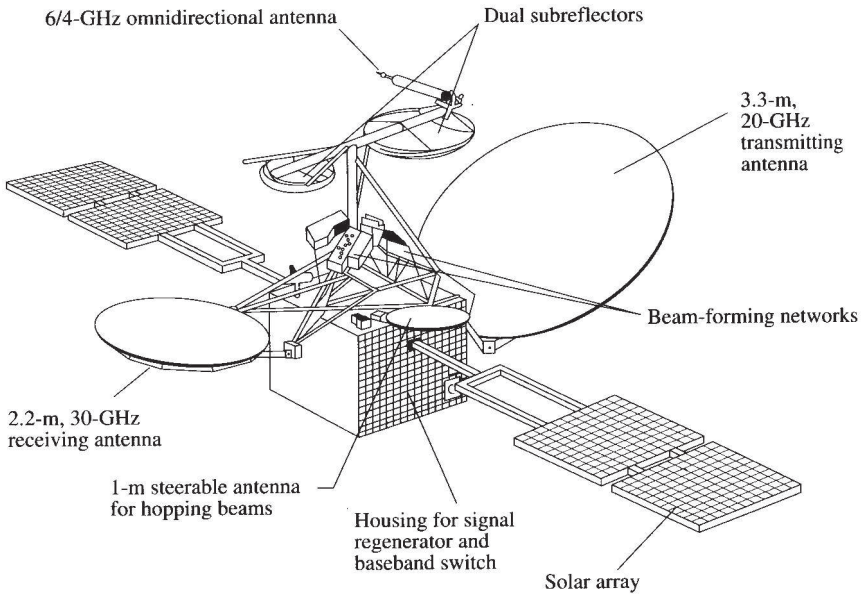


Figure 1.3 Communications satellite. (Courtesy of *IEEE Spectrum.*)

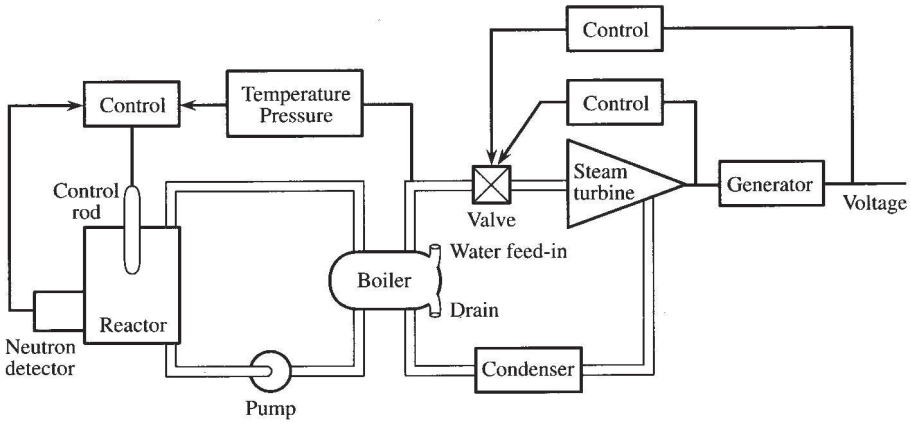


Figure 1.4 Nuclear power plant.

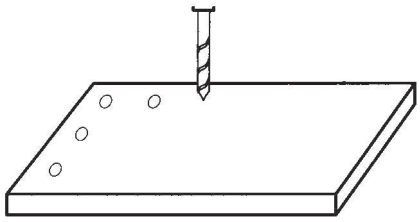


Figure 1.5 Numerical control system.

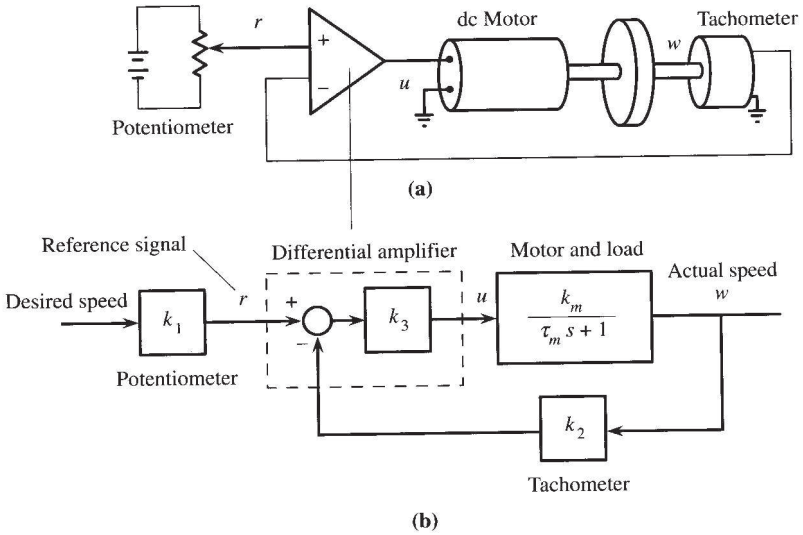


Figure 1.6 Velocity control system.

1.2.3 Temperature Control Systems

We now discuss a different type of example. Consider the temperature control of the enclosed chamber shown in Figure 1.7(a). This problem, which arises in the temperature control of an oven, a refrigerator, an automobile compartment, a house, or the living quarters of a space shuttle, can certainly be approached by using the empirical method. If the analytical method is to be used, we must develop a model as shown in Figure 1.7(b). We can then use the model to carry out analysis and design.

Temperature control is also important in chemical processes. The rate of chemical reactions often depends on the temperature. If the temperature is not properly controlled, the entire product may become useless. In industrial processes, temperature, pressure and flow controls are widely used.

1.2.4 Trajectory Control and Autopilot

The landing of a space shuttle on a runway is a complicated control problem. The desired trajectory is first computed as shown in Figure 1.8. The task is then to bring the space shuttle to follow the desired trajectory as closely as possible. The structure of a shuttle is less stable than that of an aircraft, and its landing speed cannot be controlled. Thus, the landing of a space shuttle is considerably more complicated than that of an aircraft. The landing has been successfully accomplished with the aid of on-board computers and altimeters and of the sensing devices on the ground, as shown in Figure 1.8. In fact, it can even be achieved automatically, without involving astronauts. This is made possible by the use of an autopilot. The autopilot is now widely used on aircrafts and ships to maintain desired altitude and/or heading.

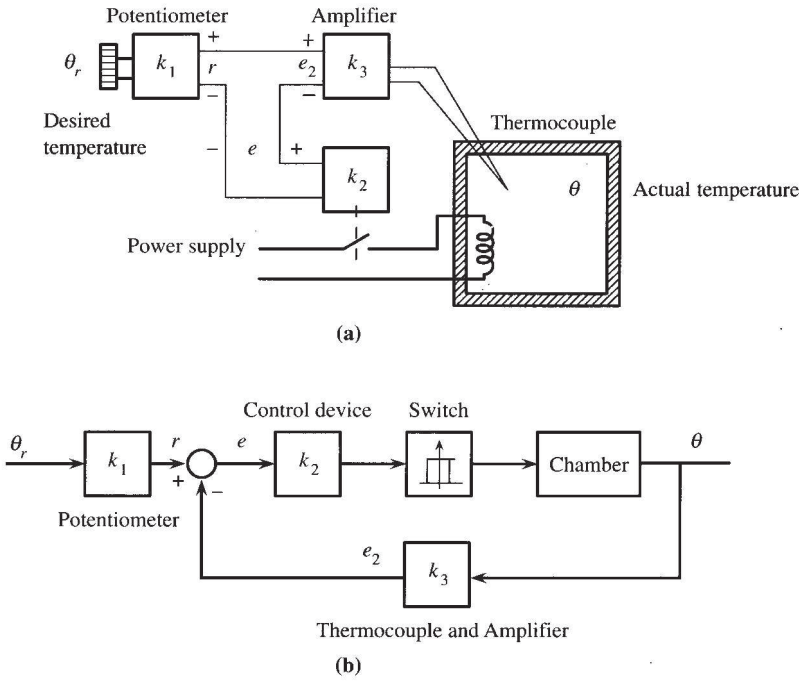


Figure 1.7 Temperature control system.

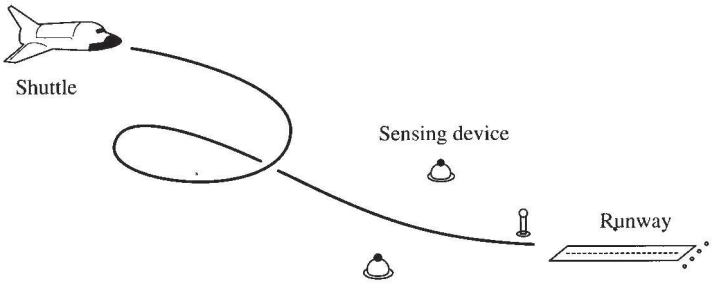


Figure 1.8 Desired landing trajectory of space shuttle.

1.2.5 Miscellaneous Examples

We give two more examples of control systems to conclude this section. Consider the bathroom toilet tank shown in Figure 1.9(a). The mechanism is designed to close the valve automatically whenever the water level reaches a preset height. A schematic diagram of the system is shown in Figure 1.9(b). The float translates the water level into valve position. This is a very simple control problem. Once the mechanism

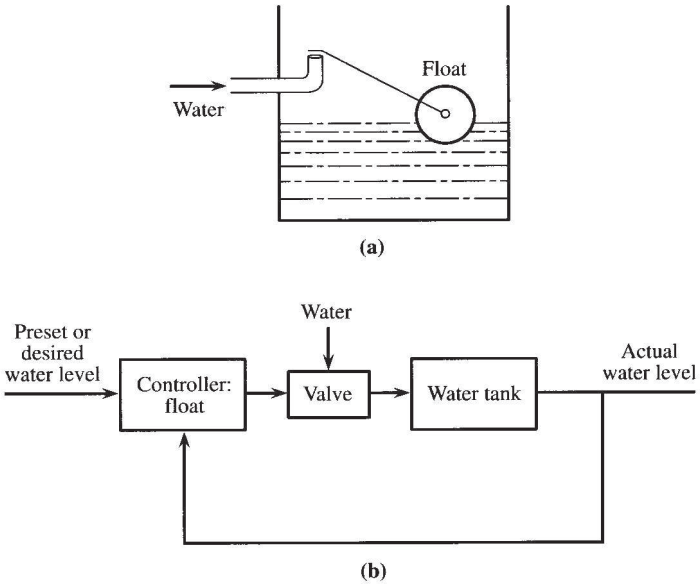


Figure 1.9 Bathroom toilet tank.

of controlling the valve is understood, the water level can easily be controlled by trial and error.

As a final example, a schematic diagram of the control of clothes dryers is shown in Figure 1.10. Presently there are two types of clothes dryers—manual and automatic. In a manual clothes dryer, depending on the amount of clothes and depending

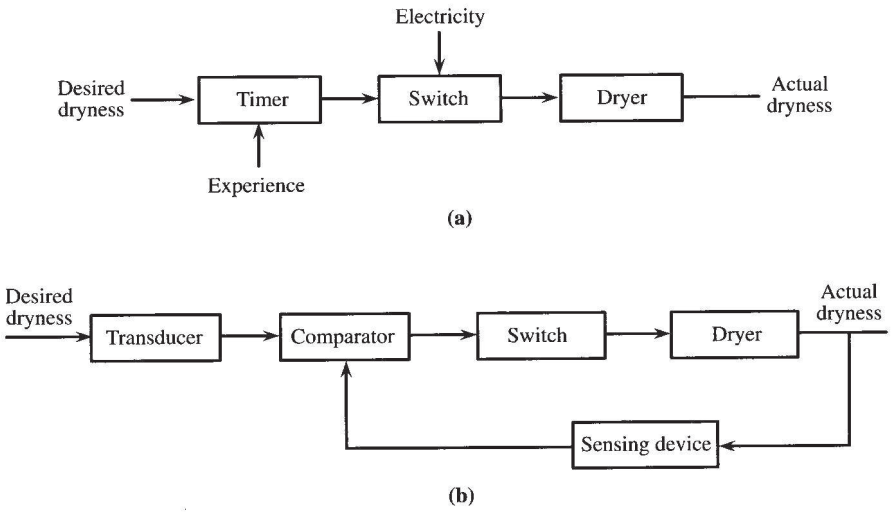


Figure 1.10 (a) Manual clothes dryer. (b) Automatic dryer.

on experience, we set the timer to, say, 40 minutes. At the end of 40 minutes, this dryer will automatically turn off even if the clothes are still damp or wet. Its schematic diagram is shown in Figure 1.10(a). In an automatic dryer, we select a desired degree of dryness, and the dryer will automatically turn off when the clothes reach the desired degree of dryness. If the load is small, it will take less time; if the load is large, it will take more time. The amount of time needed is automatically determined by this type of dryer. Its schematic diagram is shown in Figure 1.10(b). Clearly, the automatic dryer is more convenient to use than a manual one, but it is more expensive. However, in using a manual dryer, we may overset the timer, and electricity may be wasted. Therefore, if we include the energy saved, an automatic dryer may turn out to be more economical.

1.3 PROBLEM FORMULATION AND BASIC TERMINOLOGY

From the examples in the preceding section, we may deduce that a control system is an interconnection of components or devices so that the output of the overall system will follow as closely as possible a desired signal. There are many reasons to design control systems:

1. *Automatic control:* The temperature of a house can be automatically maintained once we set a desired temperature. This is an automatic control system. Automatic control systems are used widely and are essential in automation in industry and manufacturing.
2. *Remote control:* The quality of reception of a TV channel can be improved by pointing the antenna toward the emitting station. If the antenna is located at the rooftop, it is impractical to change its direction by hand. If we install an antenna rotator, then we can control the direction *remotely* by turning a knob sitting in front of the TV. This is much more convenient. The Hubble space telescope, which is orbiting over three hundred miles above the earth, is controlled from the earth. This remote control must be done by control systems.
3. *Power amplification:* The antennas used to receive signals sent by *Voyager 2* have diameters over 70 meters and weights over several tons. Clearly, it is impossible to turn these antennas directly by hand. However, using control systems, we can control them by turning knobs or by typing in command signals on computers. The control systems will then generate sufficient power to turn the antennas. Thus, power amplification is often implicit in many control systems.

In conclusion, control systems are widely used in practice because they can be designed to achieve automatic control, remote control, and power amplification.

We now formulate the control problem in the following. Consider the the position control problem in Figure 1.1, where the objective is to control the direction of the antenna. The first step in the design is to choose a motor to drive the antenna. The motor is called an *actuator*. The combination of the object to be controlled and the actuator is called the *plant*. In a home heating system, the air inside the home is

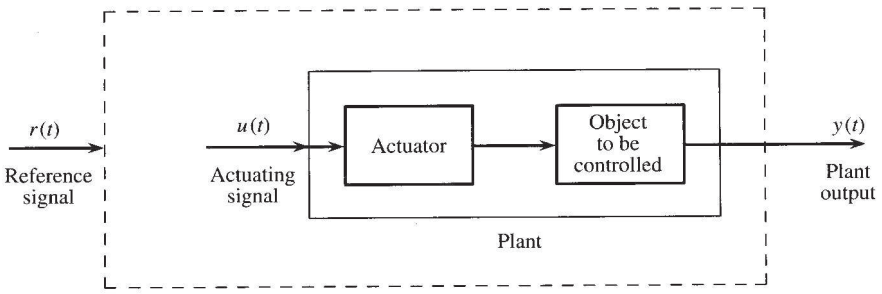


Figure 1.11 Control design problem.

the controlled object and the burner is the actuator. A space shuttle is a controlled object; its actuator consists of a number of thrustors. The input of the plant, denoted by $u(t)$, is called the *control signal* or *actuating signal*; the output of the plant, denoted by $y(t)$, is called the *controlled variable* or *plant output*. The problem is to design an overall system as shown in Figure 1.11 so that the plant output will follow as closely as possible a *desired* or *reference signal*, denoted by $r(t)$. Every example in the preceding section can be so formulated.

There are basically two types of control systems: the open-loop system and the closed-loop or feedback system. In an *open-loop* system, the actuating signal is predetermined by the desired or reference signal; it does not depend on the actual plant output. For example, based on experience, we set the timer of the dryer in Figure 1.10(a). When the time is up, the dryer will stop even if the clothes are still damp or wet. This is an open-loop system. The actuating signal of an open-loop system can be expressed as

$$u(t) = f(r(t))$$

where f is some function. If the actuating signal depends on the reference input and the plant output, or if it can be expressed as

$$u(t) = h(r(t), y(t))$$

where h is some function, then the system is a closed-loop or feedback system. All systems in the preceding section, except the one in Figure 1.10(a), are feedback systems. In every feedback system the plant output must be measured and used to generate the actuating signal. The plant output could be a position, velocity, temperature, or something else. In many applications, it is transformed into a voltage and compared with the reference signal, as shown in Figure 1.12. In these transformations, sensing devices or transducers are needed as shown. The result of the comparison is then used to drive a *compensator* or *controller*. The output of the controller yields an actuating signal. If the controller is designed properly, the actuating signal will drive the plant output to follow the desired signal.

In addition to the engineering problems discussed in the preceding section, a large number of other types of systems can also be considered as control systems.

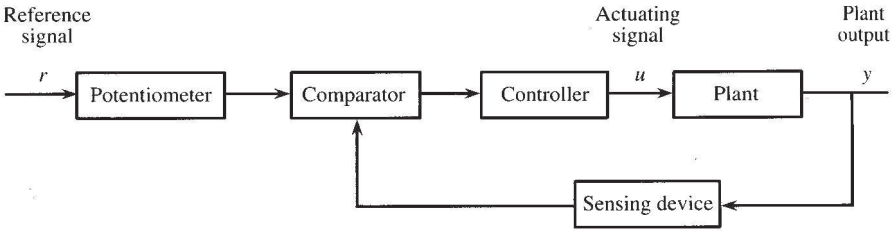


Figure 1.12 Feedback control system.

Our body is in fact a very complex feedback control system. Maintaining our body temperature at 37° Celsius requires perspiration in summer and contraction of blood vessels in winter. Maintaining an automobile in a lane (plant output) is a feedback control system: Our eyes sense the road (reference signal), we are the controller, and the plant is the automobile together with its engine and steering system. An economic system is a control system. Its health is measured by the gross national product (GNP), unemployment rate, average hourly wage, and inflation rate. If the inflation rate is too high or the unemployment rate is not acceptable, economic policy must be modified. This is achieved by changing interest rates, monetary policy, and government spending. The economic system has a large number of interrelated factors whose cause-and-effect relationships are not exactly known. Furthermore, there are many uncertainties, such as consumer spending, labor disputes, or international crises. Therefore, an economic system is a very complex system. We do not intend to solve *every* control problem; we study in this text only a very limited class of control problems.

1.4 SCOPE OF THE TEXT

This text is concerned with the analysis and design of control systems. As it is an introductory text, we study only a special class of control systems. Every system—in particular, every control system—is classified dichotomously as linear or nonlinear, time-invariant or time-varying, lumped or distributed, continuous-time or discrete-time, deterministic or stochastic, and single-variable or multivariable. Roughly speaking, a system is *linear* if it satisfies the additivity and homogeneity properties, *time-invariant* if its characteristics do not change with time, and *lumped* if it has a finite number of state variables or a finite number of initial conditions that can summarize the effect of past input on future output. A system is *continuous-time* if its responses are defined for all time, *discrete-time* if its responses are defined only at discrete instants of time. A system is *deterministic* if its mathematical description does not involve probability. It is called a *single-variable* system if it has only one input and only one output; otherwise it is called a *multivariable* system. For a more detailed discussion of these concepts, see References [15, 18]. *In this text, we study only linear, time-invariant, lumped, deterministic, single-variable systems.* Although

this class of control systems is very limited, it is the most important one. Its study is a prerequisite for studying more general systems. Both continuous-time and discrete-time systems are studied.

The class of systems studied in this text can be described by ordinary differential equations with real constant coefficients. This is demonstrated in Chapter 2 by using examples. We then discuss the *zero-input response* and the *zero-state response*. The *transfer function* is developed to describe the zero-state response. Since the transfer function describes only the zero-state response, its use in analysis and design must be justified. This is done by introducing the concept of *complete characterization*. The concepts of *properness, poles,* and *zeros* of transfer functions are also introduced. Finally, we introduce the *state-variable equation* and its discretization. Its relationship with the transfer function is also established.

In Chapter 3, we introduce some control components, their models, and their transfer functions. The *loading problem* is considered in developing the transfer functions. Electrical, mechanical, and electromechanical systems are discussed. We then discuss the manipulation of *block diagrams* and *Mason's formula* to conclude the chapter.

The *quantitative* and *qualitative analyses* of control systems are studied in Chapter 4. Quantitative analysis is concerned with the response of systems due to some specific input, whereas qualitative analysis is concerned with general properties of systems. In quantitative analysis, we also show by examples the need for using feedback and tachometer feedback. The concept of the *time constant* is introduced. In qualitative analysis, we introduce the concept of *stability*, its condition, and a method (the Routh test) of checking it. The problems of *pole-zero cancellation* and *complete characterization* are also discussed.

In Chapter 5, we discuss *digital* and *analog computer simulations*. We show that if the state-variable description of a system is available, then the system can be readily simulated on a digital computer or built using operational amplifier circuits. Because it is simpler and more systematic to simulate transfer functions through state-variable equations, we introduce *the realization problem*—the problem of obtaining state-variable equations from transfer functions. Minimal realizations of vector transfer functions are discussed. The use of MATLAB, a commercially available computer-aided design package, is discussed throughout the chapter.

Chapters 2 through 5 are concerned with modeling and analysis problems; the remaining chapters are concerned with the design problem. In Chapter 6, we discuss the choice of plants. We then discuss physical constraints in the design of control systems. These constraints lead to the concepts of *well-posedness* and *total stability*. The *saturation problem* is also discussed. Finally, we compare the merits of *open-loop* and *closed-loop systems* and then introduce two basic approaches—namely, outward and inward—in the design of control systems. In the *outward approach*, we first choose a configuration and a compensator with open parameters and then adjust the parameters so that the resulting overall system will (we hope) meet the design objective. In the *inward approach*, we first choose an overall system to meet the design objective and then compute the required compensators.

Two methods are available in the outward approach: the *root-locus method* and the *frequency-domain method*. They were developed respectively in the 1950s and

1940s. The root-locus method is introduced in Chapter 7 and the frequency-domain method in Chapter 8. Both methods are trial-and-error methods.

The inward approach consists of two parts: the search for an overall transfer function to meet design specifications and the implementation of that overall transfer function. The first problem is discussed in Chapter 9, where overall systems are chosen to minimize the quadratic performance index and the ITAE (integral of time multiplied by absolute error). It is also shown by examples that good overall transfer functions can also be obtained by computer simulations. The implementation problem is discussed in Chapter 10, where the difference between model matching and pole placement is also discussed. We discuss the implementation in the *unity-feedback* configuration, *two-parameter* configuration, and the *plant input/output feedback* configuration. We also discuss how to increase the degree of compensators to achieve robust tracking and disturbance rejection. The design methods in Chapter 10 are called the *linear algebraic method*, because they are all achieved by solving linear algebraic equations.

The design methods in Chapters 6 through 10 use transfer functions. In Chapter 11, we discuss design methods using state-variable equations. We introduce first the concepts of *controllability* and *observability* and their conditions. Their relationships with pole-zero cancellations are also discussed. We then use a network to illustrate the concept of *equivalent state-variable equations*. Pole placement is then carried out by using equivalent equations. The same procedure is also used to design full-dimensional state estimators. *Reduced-dimensional estimators* are then designed by solving *Lyapunov equations*. The connection of state feedback to the output of state estimators is justified by establishing the *separation property*. Finally, we compare the design of state feedback and state estimator with the linear algebraic method.

Chapters 3 through 11 study continuous-time control systems. The next two chapters study discrete-time counterparts. Chapter 12 first discusses the reasons for using digital compensators to control analog plants and then discusses the interfaces needed to connect analog and digital systems. We introduce the *z-transform, difference equations, state-variable equations, stability*, and the *Jury test*. These are the discrete-time counterparts of the continuous-time case. The relationship between the frequency response of analog and digital transfer functions is also discussed. Chapter 13 discusses two approaches in designing digital compensators. The first approach is to design an analog compensator and then discretize it. Six different discretization methods are introduced. The second approach is to discretize the analog plant into an equivalent digital plant and then design digital compensators. All analog methods, except the frequency-domain method, are directly applicable to design digital compensators without any modification.

If a plant can be modeled as linear, time-invariant, and lumped, then a good control system can be designed by using one of the methods discussed in this text. However, many plants, especially industrial processes, cannot be so modeled. *PID controllers* may be used to control these plants, as discussed in Chapter 14. Various problems in using PID controllers are discussed. The *Laplace transform* and *linear algebraic equations* are reviewed in Appendices A and B: this discussion is not exhaustive, going only to the extent needed in this text.